Evolutionary dynamics of any multiplayer game on regular graphs

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In the simplest approach there are only two choices (henceforth strategies) for participants what to follow in a social dilemma. In the donation game the participants (called players) can cooperate and contribute to a common income distributed equally or defect, hence just enjoy the fruit of others efforts. If we allow players to adopt the strategy of a more successful partner then the key question is what sort of conditions make possible for cooperators to survive this seemingly disadvantageous situation. In a more realistic mathematical model the participants do not interact with everyone randomly but only with a small group of participants (e.g., their neighbours) that is defined by an appropriate interaction graph. The problem becomes more subtle when there are more options (strategies) for the players. One of these systems is the so-called second-order free-riding problem, where costly punishment of defectors may support the maintenance of cooperation. Here the third strategy represents the second order free rider who is a cooperator meantime she doesn't pay punishment costs. For the three-strategy games, however, the resulting dynamics often lead to diverse and complex phenomena, such as cyclic dominance that is captured by the well-known rock-paper-scissors game.

Previous theoretical research primarily focused on two-strategy systems while the analytical solution for multi-strategy multiplayer games on graphs remained unexplored. In present work the authors solve this problem on regular graphs where the number of neighbors is fixed. Using combinatorial methods they show that the local configuration of multiplayer games on graphs is equivalent to distributing k identical co-players among n distinct strategies. They describe an discuss a general solution for any n-strategy multiplayer game when the payoff difference affect weakly the adoption of a neighbouring strategy.

This analytical method makes possible to revisit some celebrated problems where only numerical solutions are available for several interaction graphs. The introduction of the third strategy in the second-order free-riding problem cannot truly resolve social dilemmas in a community where interacting groups are formed at random. For local neighbourhood, however, this new calculation can derive an accurate threshold for the punishment strength, beyond which punishment can either lead to the extinction of defection or transform the system into a rock-paper-scissors-like cycle. The analytical solution agrees qualitatively with the prediction of numerical simulations obtained previously for non-marginal selection strengths. Similar qualitative agreement is found when other versions (e.g., pool or institutional) punishment is used to minimize defection. Their results shed light on general relationships that were indicated previously in independent research efforts obtained for weak and non-marginal selection limits.

Reference:
https://www.nature.com/articles/s41467-624-49505-5